The Pennsylvania State University

The Graduate School

Graduate Program in Acoustics

LOW-FREQUENCY CONTROL

IN SMALL, CRITICAL LISTENING ROOMS

A Paper in

Acoustics

by

Keenan A Hye

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Engineering in Acoustics

December, 2018

Keenan A Hye

We approve the paper of Keenan Hye

Date of Signature

_____________________________ _____________

David C. Swanson
Associate Research Professor of Acoustics
Paper Adviser

_____________________________ _____________

Victor W. Sparrow
United Technologies Corporation Professor of Acoustics
Director, Graduate Program in Acoustics

ABSTRACT

Though recorded music is perhaps one of the most widely enjoyed artform, it is seldom appreciated in a proper environment. Suboptimal listening environments are the status quo, and because of this, the art of recorded music cannot be appreciated to its fullest extent. Most troubling and persistent are distortions at low-frequencies, where any reasonably sized listening room will have an audible and detrimental effect in faithful reproduction of the recording. This paper seeks to aid in solving that problem by exploring various methods to enhance the low-frequency acoustic performance of small, critical listening spaces.

The paper begins with definitions of the frequency range and room size of interest. Sound wave interaction within an enclosure is discussed, and modal standing wave behavior is pinpointed as the primary acoustic distortion of music reproduction at low-frequencies. Lumped element modeling is briefly introduced to be applied to some of the low-frequency control devices in discussion, as nearly all of them transduce low-frequency energy out of the acoustic domain.

The bulk of the paper discusses various low-frequency control devices at a high level. Their design, function, and characteristics are described in the context of improving music reproduction in a space. The paper concludes with an overarching design process employing the various control methods discussed.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

Recorded music and song are overwhelmingly experienced in non-ideal listening environments. On the other hand, recorded visual artform are often experienced within dedicated viewing venues designed to maximize immersion into and impact of the piece; a bright image in darkened movie theater or a blank wall behind an oil painting act to bring complete clarity to the piece being presented. From this clarity, a more nuanced perception and understanding of the piece can be experienced. For musical artform this clarity is, unfortunately, completely absent in the typical listening experience. Meaning is obfuscated by electrical, mechanical, and acoustical distortions whose ubiquity has sadly become the norm.

Recorded music can be negatively distorted in its base digital or analog signal, in the transducer through which it is presented, and in the acoustic space through which it is finally perceived by a listener. Much focus is given to improving fidelity in the raw digital signal and the transduction process, but improvements to the listening environment itself are largely ignored. Listening to music on high-fidelity loudspeakers in a typical living space can be thought of as the equivalent to watching a movie on a high-resolution display outdoors in bright sunlight—much of the clarity of the presented artform is lost due to interference of the environment, and its full appreciation is impossible.

Fortunately, there is hope. Relatively simple adjustments to the typical room can fix most problems in most rooms at most frequencies. For instance, adding a thick carpet to a residential living space can quite seriously address many perceptible high and mid-frequency acoustic problems in the space. Other simple solutions to high and mid-frequency acoustic imperfections in rooms are abundant (see Toole, 2017). However, although simple methods can address high-frequencies acoustic concerns in rooms, low-frequency signals are much more difficult to practically control. Low-frequency sound waves often have a wavelength much larger than the room in which they are presented, resulting in significant and complex interactions between the transducer and the space. These interactions provide extreme and audible distortions to the musical signal that are impossible to mitigate with simple room finishes.

To appreciate recorded music to the fullest extent, low-frequency acoustic control must be addressed. Since most recorded music is experienced in relatively “small” rooms (from typical living rooms to studio control rooms), low-frequency problems can be diagnosed by analyzing the sound field in a typical small listening room. From there, mitigation discussions will follow.

In this paper, practical methods of acoustic control of low-frequency sound in small rooms will be outlined. In the introduction, the low-frequency, small-room sound field and associated acoustical transduction concepts will be presented. Room modes will be deemed the primary undesirable artifact of low-frequency sound reproduction in rooms. This paper’s body will then discuss both passive and active methods of controlling room mode behavior, with the goal of improving the quality of music reproduction in the space.

CHAPTER 2

LOW FREQUENCY SOUND FIELDS IN SMALL ROOMS

To understand methods of acoustic control at low-frequencies, an introduction to low-frequency sound waves and their natural action in typical small rooms must be given. Of course, then, quantification of a frequency range representing "low-frequencies" must first be addressed.

Frequency Range of Interest

Low-frequencies in this discussion will lie between a low and high cutoff-frequency that will be based on perceptual attributes to encompass all frequencies which may be thought of as "bass" frequencies by the naïve listener.

The low cutoff-frequency, f_l , is simple to define. Because average human hearing does not—on average—extend below 20 Hertz (Hz), and professionally produced music signals therefore hardly ever contain spectral content below 20 Hz, frequencies below this need not be considered in this discussion. Thus, simply, $f_l = 20\ Hz$.

The high cutoff-frequency, f_h , will be more nuanced to determine. This value is dependent on the size and finishes of the room, as well as the positions of the source and receiver within it.

The perception of "uneven bass" in a room largely depends on the density of large modal peaks in the space. At lower frequencies, the peaks are spaced far apart, and one bass note may be significantly louder or quieter than the previous note. As frequency increases, these modes "pack" closer together, and, at a certain point, become perceptually indistinguishable from one another. Modal effects are no longer perceptible for discrete bass frequencies, and are therefore much easier to mitigate using typical broadband absorption. [2].

This transition happens near what is referred to as the Schroeder frequency of the room, seen in Figure 1. The Schroeder frequency of an arbitrary room can be given by

$$f_h = 2000\sqrt{\frac{T}{V'}}$$

Where

$T = 60\ dB\ reverberation\ time\ in\ seconds\ and$

$V' = the\ volume\ of\ the\ enclosure\ in\ meters$

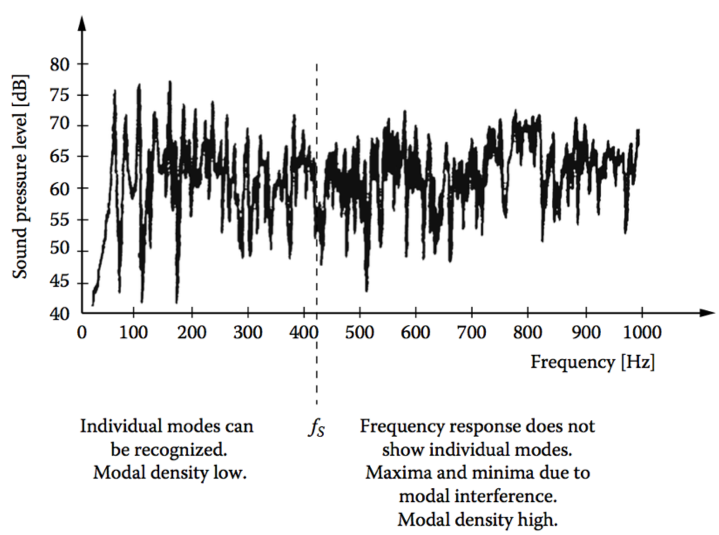

Figure 1: The Schroeder Frequency, represented as f_s

This equation is given from [3] as revised from its original value by Schroeder in [4]. It is dependent on both the room's dimensions (which define its volume) and the room's finishes (which define its reverberation time).

Though this equation was originally defined for large, diffuse spaces (both of which typical listening rooms are not), it can act as the foundation for discussion, if for nothing else than the lack of an alternative definition specific to the case of small, non-diffuse listening rooms. Baskind and Polack [5] determined experimentally that even in diffuse rooms, this higher cutoff is often misleading; since individual modal affects inherently depend on source position (to be discussed in chapter 2.2), the transition frequency will also depend on source position. In light of these concerns, it makes sense to add a margin of error to the defined high-frequency cutoff. Thus, Schroeder's original transition frequency of [4]:

$$f_h = 4000\sqrt{\frac{T}{V'}} \tag{1}$$

will be used. Seen below are a few regarded curves for reverberation times of different music studio types as borrowed from [6]:

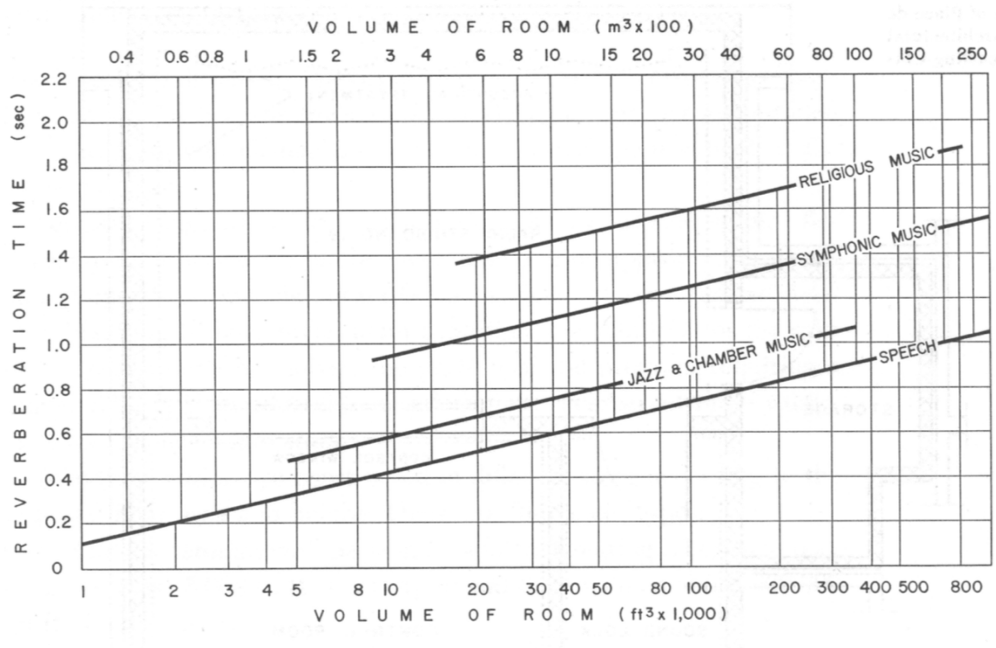

Figure 2: Common Reverberation Time Goals

The given room volume in this discussion will be under 10,000 cubic feet to be considered a “small” room. Using the “Jazz & Chamber Music” curve will give a reverberation time of about 570 milliseconds and a Schroeder frequency of about 180 Hz. Let’s say the small room also must be larger than 2,000 cubic feet. Using the same logic gives a higher Schroeder frequency of about 320 Hz, or, conservatively rounded up, about 350 Hz.

This discussion will thus restrain itself to frequencies between 20 and 350 Hertz.

The Sound Field

Now that the frequency range of interest has been defined, analysis of the sound field in this frequency range can be addressed. Since musical sounds always flow from a loudspeaker, through the acoustic space, and to the listener, there are three components of the system:

1. **the source** (in this case, a loudspeaker),
2. **the acoustic space** (in this case, a listening room), and
3. **the receiver** (in this case, a human listener)

It is important to establish a few parameters of each before discussing their interactions as they apply at low-frequencies.

A. The source:

This paper will not discuss loudspeaker design in lieu of a deeper dive into acoustics of the space itself. Instead, an idealized loudspeaker at low-frequencies will be assumed, which will be specified as having

- a **monopole**-like directivity—or equal sound power radiated in all directions, and
- a **flat sound power** spectrum—or equal sound power at all frequencies

These characteristics of the source can be quantified by first recalling the inhomogeneous Helmholtz Equation from [7]

$$\nabla^2 \hat{p} + k^2 \hat{p} = -4\pi \hat{S} \delta(\vec{x} - \overrightarrow{x_s}) \tag{2}$$

which represents the common wave equation for a point source in the frequency domain (as the delta dirac function, or a force source, in the frequency domain.) The Green's function can also be used, given by

$$G(\vec{x} - \overrightarrow{x_s}) = G(R) = \frac{e^{ikR}}{R} \tag{3}$$

as a solution, arbitrarily setting $\hat{S} = 1$. Here, $(\vec{x} - \overrightarrow{x_s}) = R$ represents the radial distance from the source to the receiver. The Green's function can be used to represent the complex pressure at a radial distance R from the source as

$$\hat{p} = \hat{S}\frac{e^{ikR}}{R} \tag{4}$$

where

$\hat{S}$ represents the complex pressure amplitude, and

k represents the wavenumber $k = \frac{\omega}{c}$, with

$\omega = 2\pi f$ as the angular frequency in radians per second and

c as the speed of sound in the propagation medium (assuming normal temperature and pressure and therefore $c \cong 343\ m/s$ throughout).

Since the source is a monopole, the pressure will only depend on the radial distance R, and not on azimuth or elevation angle. [7] Here and throughout, complex sound pressure at any given space and frequency will be notated by $\hat{p}(r, \omega)$ or $\hat{p}$ as its shorthand.

As pressure is represented by a Green's function, the principle of superposition as discussed in [8] will hold here, i.e. the effect of multiple sources in the room will be linearly additive in this discussion.

B. The acoustic space:

The acoustic space will generally be considered to be a rectangular enclosure, as nearly all listening rooms are rectangular in shape. This enclosure contains within it both the sound source and receiver, as well as 6 discrete boundaries: left, right, front, and rear walls, the floor, and the ceiling. The enclosure can be seen below with the dimensions L_x, L_y, and L_z:

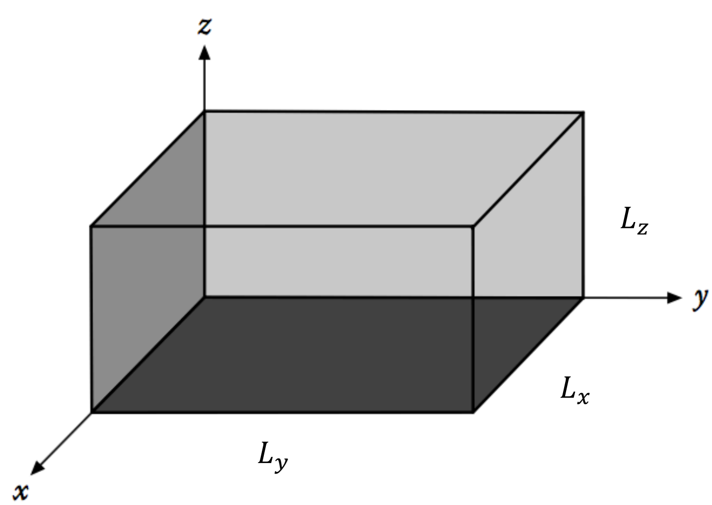

Figure 3: Dimensions of a rectangular enclosure as adopted from [2]

Each of these boundaries will, in reality, have variable acoustic impedance as a function of position on the boundary. This will be dependent on the finishes in the room and, at low-frequencies, the wallboard assembly construction. These boundary conditions will determine the shape and rate of oscillation of the room's resonances in the form of standing waves, referred to as room modes. Room modes are the primary culprit of acoustic distortions at low-frequencies in rooms, and their mitigation will be the focal point of this paper. Three types of room modes exist:

1) Axial room modes: standing waves between two room boundaries,
2) Tangential room modes: standing waves between three boundaries, and
3) Oblique room modes: standing waves between four or more boundaries

[2]. Typically, axial room modes will be most prevalent in the distortion of musical signals within the space, as they have twice the energy of tangential modes, and four times that of oblique modes [2]. Most low-frequency mitigation in the space will thus be dealing primarily with axial modes.

Theoretical room mode calculation can be simplified by setting the walls' impedance uniformly to $Z_{all} = \infty$, the impedance of an infinitely rigid boundary. With these boundary conditions, particle velocity will be at a minimum (exactly, zero) at the boundary and pressure will be at a maximum. This and the governing Helmholtz equation give the modal standing wave shapes (or eigenfunctions) via

$$\nabla^2 \hat{p}(r,\omega) + k^2 \hat{p}(r,\omega) = 0 \tag{5}$$

with $\nabla \hat{p}(r,\omega) \cdot \vec{n} = 0$ at the boundaries, where $\vec{n}$ is the normal vector to the boundaries.

It can be shown using the method of separation of variables that if pressure consists of a multiplication of axial components, $\hat{p}(r,\omega_n) = \hat{p}_x(x,\omega_n)\hat{p}_y(y,\omega_n)\hat{p}_z(z,\omega_n)$, then the Helmholtz equation would take form

$$\frac{\hat{p}_x''}{\hat{p}_x} + \frac{\hat{p}_y''}{\hat{p}_y} + \frac{\hat{p}_z''}{\hat{p}_z} + k^2 = 0 \quad (6)$$

The Helmholtz equation $\nabla^2 \hat{p}_s + k^2 \hat{p}_s = 0 \rightarrow \frac{\hat{p}_s''}{\hat{p}_s} = -k_s^2$ for any axis $s = x, s = y, s = z$, then give $\hat{p}_s'' + k_s^2 \hat{p}_s = 0$ for each axis s, which has the solution

$$\hat{p}_s = a\, \mathrm{e}^{\mathrm{i}k_s s} + \mathrm{b}\, \mathrm{e}^{-\mathrm{i}k_s s} = \quad a\cos(k_s s) + b sin(k_s s)$$

for each dimension s. The boundary conditions can now be applied to first find the eigenfunction per axis $\nabla \hat{p}_s = \mathrm{a}\cos{(k_s s)}$ via

$$\nabla \hat{p}_s \cdot \overrightarrow{n_s} = \left.\frac{\delta \hat{p}_s}{\delta s}\right|_{s=0} = 0 \rightarrow -ak_s \sin(k_s 0) + bk_s \cos(k_s 0) = 0 \rightarrow b = 0 \rightarrow$$

$$\nabla \hat{p}_s = \mathrm{a}\cos(k_s s) \quad (7)$$

And then the eigenvalues per axis $k_s = \frac{n_s \pi}{L_s}$ via

$$\left.\frac{\delta \hat{p}_s}{\delta s}\right|_{s=L_s} = 0 \rightarrow -ak_s \sin(k_s L_s) = 0 \rightarrow k_s L_s = n\pi \rightarrow$$

$$k_s = \frac{n_s \pi}{L_s} \quad (8)$$

Now, pressure can be expressed as a multiplication of these functions per dimension scaled by a certain amount at each modal index n_x, n_y, n_z:

$$\hat{p}(r, n_x, n_y, n_z) = A_{n_x,n_x,n_z} \cos\left(\frac{\pi n_x}{L_x} x\right) \cos\left(\frac{\pi n_y}{L_y} y\right) \cos\left(\frac{\pi n_z}{L_z} z\right) \tag{9}$$

It is simple then to find that the corresponding angular frequencies for these modes are given by

$$\omega_{n_x,n_y,n_z} = c\sqrt{\left(\frac{\pi n_x}{L_x}\right)^2 + \left(\frac{\pi n_y}{L_y}\right)^2 + \left(\frac{\pi n_z}{L_z}\right)^2}$$

Or in terms of normal Hertz, the oft cited equation

$$f_{n_x,n_y,n_z} = \frac{c}{2}\sqrt{\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2} \tag{10}$$

These frequencies of the enclosure's room modes are the eigenvalues in the system [8]

Room modes are resonances of the system, meaning that they exist at the frequencies where impedance of the system is at a local minimum. For any resonance, frequencies on or near the resonant frequency will have an extended decay or "ringdown" time. The sharper the modal peak in the frequency domain (or the larger its Q), the longer its perceived decay [9]. This is another deficiency provided by room modes which negatively affects transient low-frequency signals (like a kick drum.)

With non-rigid walls, and thereby complex, finite, and spatially-variable boundary impedance, finding resonant frequencies of the system naturally becomes more complicated. Pressure maxima (and particle velocity minima) shift in a frequency dependent manner and will no longer exist exactly at the boundaries. This in turn alters the theoretical standing wave pattern in space and its resonance in frequency. Essentially, the role of all the acoustic devices in this paper is to exploit this shift in boundary impedance, engineering them precisely to shift standing wave patterns or reduce them in amplitude in such a way as to improve the sound field in the listening area.

There are methods to analytically solve for pressure response given arbitrary and spatially variable boundary conditions, as shown in [10], and also given by [11]. Walker importantly gives

$$p_r \approx \frac{\rho c^2 Q_0}{V} e^{-i\omega t} \sum_n \frac{\epsilon_{n_x}\epsilon_{n_y}\epsilon_{n_z}\Psi_\mathrm{n}(S)\Psi_\mathrm{n}(R)}{\frac{2\omega_n k_n}{\omega} + i\left(\frac{\omega_n^2}{\omega} - \omega\right)} \tag{11}$$

Where

Q_0 is the volume velocity of the source,

ρ is the density of the medium (at normal temperature and pressure),

c is the soundspeed of the medium,

V is the volume of the room,

ω is the angular frequency,

ω_n is the angular natural frequency of room mode n,

$\epsilon_0 = 1, \epsilon_1 = \epsilon_2 = \epsilon_3 = \cdots = 2$ is a scale factor for each mode n

$k_N = \frac{c}{8V} \cdot \frac{(\epsilon_{n_x} a_x + \epsilon_{n_y} a_y + \epsilon_{n_z} a_z)}{2}$ is the damping factor, which shall be assumed to be entirely real and based off of any given boundary impedance,

$a_s = S_s \bar{\alpha}_s$ for any given axis $s = x, s = y, s = z$ where

S_s is the total surface area of the room boundaries perpendicular to the s-axis, and

$\bar{\alpha}_s$ is the average absorption coefficient of the room boundaries perpendicular to the s-axix.

Both functions $\Psi_{\text{n}}(S)\Psi_{\text{n}}(R)$ are functions of the positioning of the source and receiver inside the room, given by

$$\Psi_n(\xi) = \cos\left(\frac{n_x \pi x_\xi}{L_x}\right)\cos\left(\frac{n_y \pi y_\xi}{L_y}\right)\cos\left(\frac{n_z \pi z_\xi}{L_z}\right) \tag{12}$$

For either the source or receiver positions in terms of cartesian coordinates relative to the walls. Note that this is the eigenfunction from (9).

Despite the complexity of these equations, these walls are still simplified to have completely real impedance and one average absorption value across its entire face. It must be kept in mind that true boundary impedances will be both complex and variable across their face.

Alternatives exist to analytical analysis. Often, Finite Element Modeling (FEM) is employed to simulate sound fields in the general impedance case. Despite lengthy computation time even on today's computers, these models are well trusted and easier to wield than analytic equations, making them a critical tool in low-frequency analysis. Measurements can also be made to observe problems and remedy them in-situ, when the chance is available. Conclusions made from proper measurements of course always trump predictive modeling, since they represent acoustic quantities exactly as in reality. Ideally, proper measurements of the sound field as it exists should be utilized for design decisions when possible.

C. The receiver:

Finally, consideration the perception of low-frequency acoustic signals by the receiver—a human listener—must be taken. A few things can be assumed:

- Like the source, the receiver will have a **monopole**-like perception of low-frequency signals—that is, no discrimination of direction at low-frequencies at all. This means that a low-frequency sound will be perceived as “directionless” by a listener, and in a small room like this an assumption like this is not far off. [12] discuss the significant lack of evidence of a human listener’s ability to determine location of low-frequency sound sources within a small room. This assumption will be adopted and discussion of optimal low-frequency radiation direction from a sound source will be forgone.
- Again, like the source, the receiver will prefer a **flat sound pressure level** per frequency—that is, a listener’s desire to hear all low-frequency signal with the same amplitude relative to each other. Should a different frequency response be desired, any desired target curve can be applied through other means (e.g. digital global equalization).

Because of the equivalent characteristics of the source and receiver, the reciprocity relation as discussed in [8] will apply in the discussion. This means that source and receiver can here be interchanged and the same sound pressure response at the receiver will hold.

With a spectrally flat source and a desired spectrally flat response at the receiver, the frequency response of the transfer function from source to receiver in the room should be, ideally, equal in magnitude at all frequencies. Of course, as discussed, room modes make this a very difficult feat to accomplish. The focus of this paper will be on various practical control methods to improve the low-frequency response within small rooms.

Of course, listening is best enjoyed with company. Because of this, consideration of the position of a few receivers within the space, or a general area in which a receiver could be located, is warranted. Since listener height is constant or generally constrained during listening, the listening area can be thought of as a plane in three-dimensional space.

CHAPTER 3

LUMPED ELEMENT CIRCUIT ANALYSIS

Many of the acoustical devices to be discuss, if not all of them, require some transduction between domains to assist in their goal of room-mode mitigation. This is because acoustic absorption in its typical form of porous materials does not work well at low-frequencies. Thus an absorption method in another domain is usually employed, usually in the mechanical or electrical domain. It is then important to establish methods to model transduction between these three domains.

Lumped element and circuit analysis can be combined to achieve this goal effectively at low-frequencies. As can be seen via the below table, there is a direct relation between potential energy, kinetic energy, displacement, and impedance between the three domains:

Domain	Potential Energy	Kinetic Energy	Impedance
Acoustical	Pressure, p	Volume velocity, U	p/U
Mechanical	Force, F	Velocity, v	F/v
Electrical	Voltage, e	Current, i	e/i

Table 1: Potential energy, kinetic energy, and impedance in the acoustical, mechanical, and electrical domains

Impedances include a resistive component R and reactive component X in each domain, in the form $Z = R + jX$, with Z being complex and j representing the imaginary number $j = \sqrt{-1}$. In the electrical domain, resistance represents resistance, and reactance represents the combined effects of inductance and capacitance. Using circuit analysis, these three typical device representations in the electrical domain can be shown to have direct analogs in other domains:

- **Electrical domain: Resistance**
    - Mechanical domain: damping or frictional losses
    - Acoustical domain: absorption loss or radiation out of the system
- **Electrical domain: Capacitance**
    - Mechanical domain: compliance (springs)
    - Acoustical domain: compliance of "compressible"* air volumes (acoustically large cavities relative to wavenumber)
- **Electrical domain: Inductance**
    - Mechanical domain: inertance (mass)
    - Acoustical domain: inertance of "incompressible"* air volumes (acoustically small cavities relative to wavenumber)

A classic example—and one which will here be explored—is taking the Helmholtz resonator from the acoustical domain and representing it in circuit form, as demonstrated in [13]:

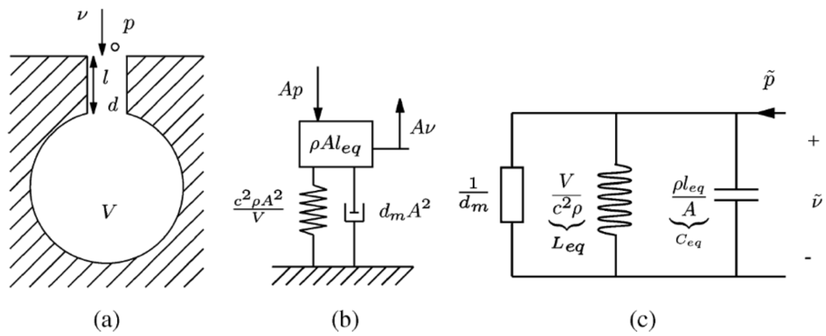

Figure 4: A Helmholtz resonator in the acoustic (a), mechanical (b), and electrical (c) domains

In the mechanical domain, the neck of the Helmholtz resonator can be thought of as a mass ($\rho A l_{eq}$), the cavity as a spring ($c^2\rho A^2/V$), and the viscous losses at the neck as a resistance ($d_m A^2$). Depictions can be seen in (b), with the kinetic energy of the velocity of the mass (A_v) being reciprocated by both damping and spring-like effects of the resonator.

Transformers serve to transduce energy between domains and correspond simply to a scale factor being applied to the potential energy or flow through a certain junction of the transduction system. Here it can be seen that in the area of the neck of the resonator being a scale factor between the pressure (potential energy in the acoustic domain) and force (potential energy in the mechanical domain), since of course $F = pA$.

Transduction in low-frequency control methods is common, and thus these concepts will return as they apply to some of the acoustical devices in this analysis.

*compressible and incompressible are here in quotes because we know that, of course, all air is compressible. However, in lumped element analysis we can effectively approximate that smaller air cavities will often act more as incompressible than compressible

CHAPTER 4

ROOM DIMENSIONS AND CONSTRUCTION

If optimization of acoustic performance at low-frequencies in a small room is desired, optimizing the room itself must first be considered. If there is any availability to design what is often called the "shell" of the room, an optimization of the room's dimensions and boundaries for low-frequency performance will, in theory, exist. As stated previously, the dimensions and boundary impedances of the space will determine the modal frequencies within the space.

Ratios of Room Dimensions

Much has been theorized on optimizing room dimensions in critical listening spaces to produce ideal modal conditions. It is thought best to begin with modal conditions that, assuming perfectly rigid walls, would give us the flattest modal frequency response. The resulting "ideal" ratios of room dimensions—room ratios—are a topic of much contention, and many possible candidates have been proposed. A few noteworthy examples can be seen in the table below.

Year	Name	Ratio	Criteria	Source
1950	Knudsen, Harris	1:1.251.6		[14]
1964	Bolt	Range (see Figure 5)	Looked for minimal equidistant spacing between modes	[15]
1971	Louden	1:1.4:1.9	Investigated 125 combinations of room dimensions, found the flattest	[16]
1981	Bonello	Variable within criteria	Each third-octave band should contain within it more modes than the previous band	[17]
2004	Cox, D’Antonio & Avis	Variable within criteria	Found flattest modal responses of many rooms via computer aided modeling	[18]

Table 2: Notable proposed ideal room ratios from various decades

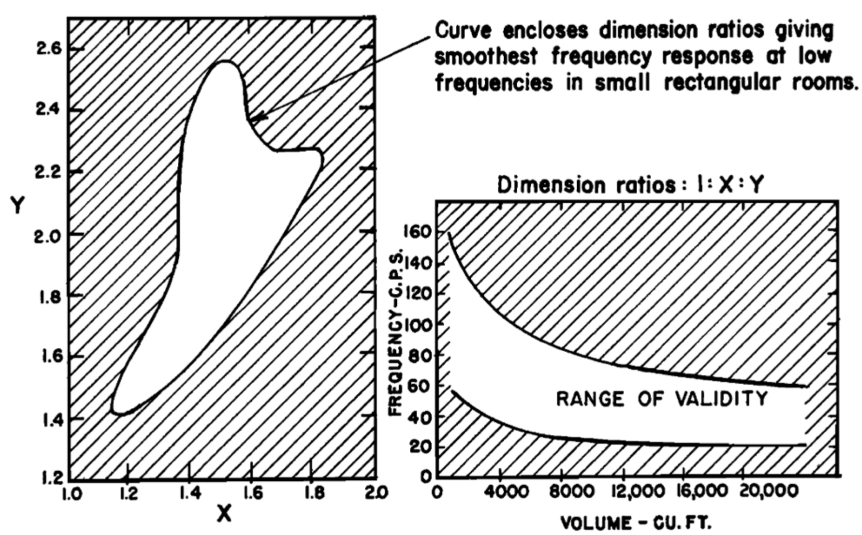

Figure 5: The range of room modes with most regular frequency spacing (left) and their range of validity (right) from [15]

Particularly notable are the relatively modern efforts of Cox, D'Antonio & Avis in 2004. Unlike previous efforts which assume infinitely rigid walls, Cox et al use the image source method with walls having variable surface reflection coefficients, allowing them to model rooms with absorption on its boundaries. They use this method to improve iteration speed and concede that modal decomposition with wall impedance—as in (11)—would yield a more accurate result should data on typical wall impedances be available. They also note that though FEM modeling would yield the most accurate results, the time it takes to run a simulation would make their iterative process infeasible. [18] Even in this more refined method the concessions are clear; there lacks a completely accurate estimation of true modal conditions.

There is a competing notion that the search for "ideal" room modes has been given much too much importance, is futile, or sits on top of logically flawed foundation. Most of these ideal ratios stem from the argument that modal regularity or their increasing density in frequency is preferable. Surprisingly, however, there exists scant subjective evidence to support a general preference of listeners for modal regularity. Some contend that due to this lack of evidence, the search for ideal room ratios is without basis and therefore futile. Toole made a convincing argument for this [1] and further explained how the ideal room ratio debate has its origins not in critical listening spaces, but in reverberation chamber design. He argued that there is no evidence to suggest room mode distribution matters, citing lack of reliable psychoacoustical evidence and the influence of defects or additional surfaces to the boundaries of the space, altering the theoretical room modes from those of reality.

Irregularly Rooms Shapes

As the room in this discussion is a rectangular enclosure, it is important to quickly note that other room shapes can be beneficial to low-frequency sound fields within a room. If there exists no parallel room boundaries, axial modes will not exist in the space. The complexity of these spaces leads to equally complex modal conditions within the space that are outside of the scope of this paper.

Coupled Cavities

Coupled cavities, volumes, or rooms can also be beneficial in their propensity to act as a sink for low-frequency pressure. As low-frequency energy is in general steered towards the cavity—or into an adjacent room—it can be simultaneously steered away from the listener (provided the listener is not sitting within the cavity.) Cavities of small volumes within the space can then additionally act as an efficient location for absorptive devices that absorb most efficiently under high-pressure conditions. Coupled cavities of large volume, such as adjoining rooms, can act as low-frequency sinks as some low-frequency energy propagates out of the room rather than resonating with the boundaries of the space. Simply opening a door can substantially improve the low-frequency performance of a room, provided, again, that sound isolation is not of great concern.

Room Construction and Wall Assemblies

As wall assemblies largely dictate the low-frequency boundary conditions of standing waves within the space, their construction is a vital component of any listening room design process. Although double-walled assemblies common to the “room-inside-a-room" design ideals of most studios and listening room are great for sound isolation [19] and bring us close to a perfectly rigid boundary ideal, their rigidity inherently leads to little low-frequency energy being absorbed at the boundaries or radiated out of the room. Although sound isolation is often an important component of any listening room design, reduced magnitude of low-frequency modes (and thus a flatter room frequency response) can be achieved with a more transmission-prone wall assembly, such as a simple single sheet of gypsum board on wood studs. This makes sense intuitively, since the less sound energy stays inside the room, the less sound energy exists for standing waves. The wallboard provides a mass, and the air cavity behind it a compliance which establish certain resonances of transmission in the assembly. Those resonances are compounded by resistance in

- frictional heat losses from the bending of the wall board,
- acoustical losses of the air molecule friction against any fiberglass in the assembly, and
- acoustical viscous losses through any small hole "leaks" such would occur at an unsealed section of or small hole in the wall.

Although this system could be modeled to predict its resonant frequency via constructing a lumped-element circuit representation (as performed in Chapter 2.3 for membrane absorbers), wall construction is a highly variable process and attempts to predict the loss and compliance of the system would be in vain, even using finite element computer modeling software. "Tuning" a wall to absorb problematic room modes is not practical, nor necessarily desirable if sound isolation is a concern. Designing rooms for high sound isolation is directly at odds with designing them for high low-frequency energy loss via the room boundaries [1]. One goal must then be deemed more important than the other while designing the space, and sound isolation usually wins out, since increased isolation give better separation from noise sources in adjacent rooms.

CHAPTER 5

LISTENING AREA, SPEAKER PLACEMENT

After the room has been constructed, consideration of the optimal placement of the sources and receiver within the space must occur. These will both of course determine the final transfer function from source to listener through the space.

Source Location

Though they are inherent to the room, room modes must be excited to appear [1]. The location of the sound source within the space will dictate which room modes exist in the space. Similarly, the location of the receiver will dictate which room modes are heard by the listener. If either sits on a natural room mode pressure maxima, that mode will be clearly perceived. Expanding this further, and adding another source at another location, different standing wave would be accentuated corresponding to the position and relative phase of both sources—there will be discussion on multiple sources in more detail in Chapter 3.1.

The first consideration while placing sources should be the goal of that placement. Which room modes should be excited? Which should be avoided? These can be controlled by the positioning of the source within the space. If constrained to placement along a wall, two extreme design processes could go as follows in a room with perfectly rigid boundaries:

- **Goal:** Excite as many room modes as possible
    - **Achieve by** placing sources at the exact intersection of three wall planes, or in other words, at the exact pressure maxima for all possible axial room modes
- **Goal:** Excite as few room modes as possible
    - **Achieve by** placing sources at the exact center of three wall planes of different axes, or in other words, at the pressure minima for all possible axial room modes

A benefit of placing sources in the corners of rooms is that their sound power output is maximized—all theoretical room modes are excited, increasing modal density and overall radiation energy transfer [8]. This may lead to less low-frequency variance, as postulated previously to be beneficial (though this is not necessarily proven subjectively.) The caveat of this method is of course that perhaps there are particularly damaging room modes that would be excited via this placement, accentuating the worst of the room.

The benefit of placing sources on the center of walls is in exciting null points in the pressure response such that they are still minima but no longer nulls. This of course helps improve the transfer functions on-axis to the walls by alleviating nulls at the listening position—likely to also be centered along one or two axes of the room. This change in sound field due to movement toward a null is depicted in the following figure from [1]:

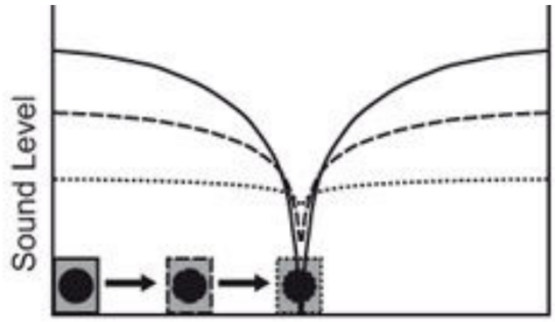

Figure 6: Modal sound field change as a function of loudspeaker distance from a boundary. The magnitude of the room mode becomes less extreme with closer placement to a null.

More commonly, a balance of the two extremes is necessary, and is dependent on the desired receiver locations. Using an iterative measurement and analysis method can flatten the frequency response considerably if movement of the source is available. [20] used a method process as follows:

1. Place source in position
2. Measure frequency response
3. If desirable frequency response achieved, **end**, if not, continue
4. Identify undesirable modal effects
5. Determine new position to move loudspeaker closer to or farther from modal pressure maximum, depending on desired change in response
6. Return to step 1

With enough time and patience, this method, can yield great improvements in the frequency response as heard in the listening position. Even with just a few repositionings, Groh demonstrated a significant improvement of frequency response. However, this becomes more complicated and infeasible with multiple sources. The distribution of multiple low-frequency sources can often yield a flatter frequency response with careful implementation (this is discussed in chapter 3.1 of this paper.)

First reflections from the planar boundaries of the room will also affect the sound heard at low-frequencies. If the source is placed at a distance from a rigid wall, sound radiating to towards the wall will reflect back to the source to positively or negatively interfere with the wave at the source position. Frequencies with a quarter-wavelength corresponding to the distance from the source to a perfectly rigid wall would be completely canceled out. This is obviously to be avoided. The quarter-wavelength of 350 Hz is about 9.6 inches, so it would be prudent to place the source at least this close to a boundary to minimize the boundary effect. Obviously, perfectly nestled into and baffled by the boundary is preferred when the option is available, as some destructive interference will still occur inside the 9.6 inch limit. However, practically, keeping the low-frequency source as flush to a wall as possible is often a reasonable and easy-to-follow design practice.

Receiver Location

Once the position of the sources are chosen, the receiver would, ideally, be able to move around the room with no change in spectral perception. Similarly, multiple receivers at multiple positions in the room would ideally have the exact same transfer function. Though of course this is known to be impossible, it is desirable to strive for as little variance in frequency response across possible listening areas to be achieved. There are three reasons for this:

1. Similar listening experience regardless of seating choice
2. Similar listening experience for all listeners in the space
3. Any equalization will affect all seats similarly (more on this in Chapter 3.2)

The placement of the listener is directly analogous to that of the placement of the sources. A listener should not be placed on nulls or peaks in the pressure response in the room, but at points of minimal modal interference. A poor choice of listener location in a room with rigid boundaries might be, for example, the dead center of the space, since this is where many modes will have their nulls [21].

If the sources are already positioned, the space can either be modeled or measured as built to decide a listening position. With the constraint of a fixed height (the height of the seated listeners), optimization of the listening position for low-frequencies would consist of finding the point on the measurement or modeling plane with the least-variant frequency response. This would of course have to be judged together with both feasibility and listening concerns for high frequency sound. An example of this—although outside the scope of this text—might be that the listener would likely want to be placed on the centerline of the room to minimize dissonant high-frequency early reflections for localization and imaging purposes. That would be another constraint to the possible listener locations in the modeling or measurement scheme.

In the author's experience, it is often a very educational exercise to experiment with ideal low-frequency listening positions in-situ by listening to challenging low-frequency program material or test tones and moving about the space at listening height. Bass-challenging program material, and a wide variety of it, should be used. It is in this way very easy to eliminate certain listening positions as non-viable, and hone-in on better-sounding positions for further objective measurement scrutiny. Of course—knowing the human mind's inherently short acoustic memory—one must remain humble to fact during this exercise. Opinion should always be verified with objective evidence of improvement, and objective improvement should always in turn be subjectively evaluated.

Software solutions

Of course, finite element modeling or simpler software can be used to virtually place a source and receiver or listening plane within a space and gauge its frequency response for preferred modal conditions. Logistically reasonable positions or ranges for sources can be used to calculate multiple options and choose the best modeled frequency response. Specialized software also exists for optimizing these locations, as well as room dimensions and treatment areas, under certain costs restraints [22]. When the opportunity is given, it only makes sense to model the space to analyze variable source and receiver placement.

CHAPTER 6

MEMBRANE ABSORBERS

Although they do not have to be, it is easiest now for us to consider the physical dimensions and positioning of the source, space, and receiver as fixed and optimized for a room with typical furnishings. Exploration of the abilities of low-frequency specific acoustic devices to tame the many remaining resonant frequencies in the transfer function between source, room, and receiver would then follow.

Porous materials are typically used as acoustic treatments in rooms to quell undesirable high and mid-frequency of the space. The air within porous materials vibrates sympathetically with high particle velocity waves, dissipating energy via frictional heat in the open cells of the material [23]. Thus, they are only practically effective in regions with high particle velocity—which does *not* occur near the wall for low-frequencies. Porous materials work well for high and mid frequencies because they usually have such small wavelengths that a particle velocity maxima usually exists somewhere close to the room boundaries. Porous material can be increased in thickness or be mounted slightly off of the wall to increase absorption at lower frequencies. However, very low frequencies have such large wavelengths that particle velocity maxima are very far away from the wall, and therefore any raw porous treatments would need to be so thick or far from the wall that their use becomes infeasible.

Thus, there is a need to turn to less typical acoustic treatments. The first of these low-frequency-specific acoustical control devices is the membrane or diaphragmatic absorber.

Membrane absorbers are rectangular enclosures with 5 faces of rigid construction and one face a limp mass membrane (or a “diaphragm”), such as mass-loaded vinyl. Inside of the air cavity sits a layer of porous absorbent material to act as the loss mechanism. Membrane absorbers sharply absorb sound at their resonant frequency and largely reflect sound at most other frequencies, making them ideal candidates for precise mitigation of room modes. They also, importantly, work best in local *pressure* maxima, making them very effective mounted flush to room boundaries [23].

Membranes in general act to transmit and reflect high pressure at variable frequency. The variable tension of the membrane coupled to a cavity volume behind allow them to be “tuned” to absorb sharply at a given frequency through iterative measurement and analysis. Sound energy must first be transmitted through the membrane, then absorbed by the absorptive filling to be reduced upon reflection back out of the membrane. Ingard described the transmission characteristics of a stretched membrane in [24]. In a general sense, membrane absorbers will have high absorption at resonant frequencies, and close to zero absorption at antiresonances. This behavior can be seen in the frequency response of the circular membrane discussed by Ingard, shown below:

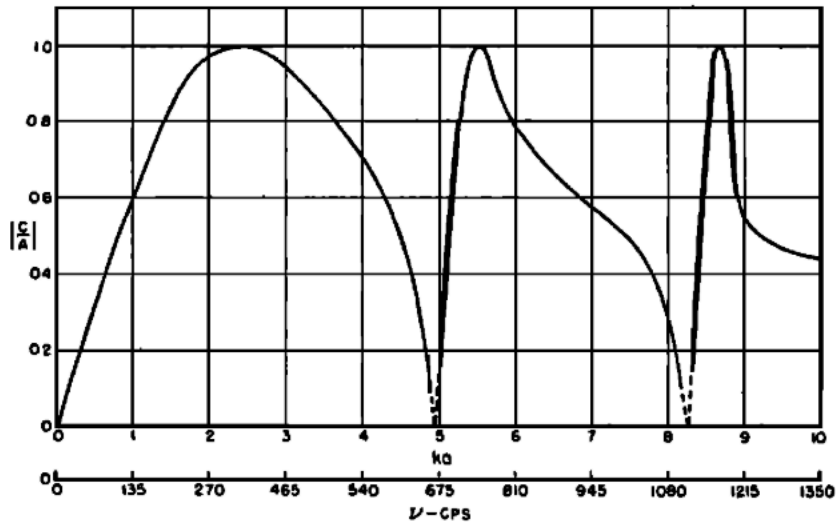

Figure 7: Transmission characteristic of Ingard's theoretical stretched circular membrane. Resonant peaks (maximum absorption) and nulls (zero absorption) can be seen in its response.

The impedance of a membrane absorber can be approximated in a similar manner as with wall assemblies in Chapter 2.1. Membrane absorbers consist of the mass of the diaphragm, the compliance of the air cavity behind it and at the edges of the diaphragm itself, and an acoustic energy loss in the porous absorbent layer inside of the enclosure (the friction of the air molecules moving through the porous layer.) Though they certainly exist, energy losses at the boundaries of the diaphragm due to friction, as well as viscous losses within the cavity of the absorber, will both be negligible enough relative to the losses from the porous material to be omitted from the model.

To further simplify the model, the incidence of a plane wave at a normal direction to the diaphragm of the membrane absorber will be the only scenario considered. Since room modes standing waves have wavelengths inherently larger than or comparable to the room enclosure dimensions, the simplification to plane waves is appropriate. With these constraints, the membrane absorber can be equivalently represented as a lumped element system with the following generalized impedance as given in [2]:

$$Z_a = R_a + j\omega M_a + \frac{1}{j\omega C_a}$$

Where R_A, M_A, C_A are acoustic resistance, acoustic mass, and the acoustic compliance of the system. These can be further represented by the following equations:

$$M_a = \frac{M}{S^2}$$

$$C_a = \frac{Sd}{\rho c^2}$$

Where

M is the mass of the membrane,

S is the surface area of the membrane, and

d is the depth of the air behind the membrane. This together gives

$$Z_a = R_a + j\omega \frac{M}{S^2} + \frac{\rho_0 c^2}{j\omega Sd} \tag{12}$$

which can be found to have a resonant frequency at its minimum given by

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{\rho}{md}} \tag{13}$$

Here will lie the absorptive peak of the membrane absorber, which can be tuned to an axial mode and placed at a maximal pressure point of that axial mode (i.e. on one of the boundaries of the offending axial mode.) The membrane absorber will then absorb some of that frequencies' amplitude and reflect it at a different phase, shifting the modal shape to look as if that dimension had been lengthened [1].

Although membrane absorbers can be modeled theoretically, it is unfortunately difficult to reliably predict their resonant frequencies precisely as they are actually built.

Cox and D'Antonio show inconsistencies between predicted and actual measured membrane absorbers [23]. A membrane absorber thus should be measured for its impedance and tuned empirically to work as designed. Should it be tuned to a measured, offending axial mode in the space, and placed on a boundary of that mode, it should absorb a high level of sound strongly at that modal frequency, while reflecting sound at other frequencies.

CHAPTER 7

HELMHOLTZ AND OPEN AREA ABSORBERS

Helmholtz Absorbers

Helmholtz absorbers are another conceptually simple passive low-frequency control device. These are essentially a large volume that generally acts as an acoustic "spring" coupled to a neck (or pipe) which generally acts as an acoustic "mass". A circuit representation for the Helmholtz absorber was derived in Chapter 1.3. These would have a resonance at the frequency which minimizes impedance. This resonance can be reduced in amplitude by adding absorptive material to the volume, with the acceptance that this will inherently change—specifically, lower—the resonant frequency of the system in the process.

As with membrane absorbers, a Helmholtz absorber could be matched to a room mode and placed where the highest pressure amplitude at that frequency would be expected: on a boundary normal to direction of the offending axial mode.

[25] detailed the design of Helmholtz resonators by first giving one with small dimensions relative to wavelength, which is appropriate in the present case of large wavelengths.

The absorption of the Helmholtz resonator in a wall at resonance was given to be

$$\sigma_a = 2\frac{R_i}{(R_i + R_r)^2}A \tag{14}$$

Where

R_i is the internal frictional resistance of the neck,

R_r is the radiation resistance of the aperture, and

A is the cross sectional area of the aperture.

He then gives the resonant frequency of the absorber to be

$$f_0 = \frac{c}{2\pi}\left[\frac{A}{V(t+\delta)}\right]^{\frac{1}{2}} \tag{15}$$

Where

c is the soundspeed of the medium

A is the area of the aperture

V is the volume of the cavity

t is the length of the neck, and

δ is the end correction to the neck length, with

$\delta = 0.96A^{\frac{1}{2}}$ given as a general rule approximation for an end correction for an arbitrarily shaped aperture of area A baffled by an infinite plane. However, this approximation only holds for necks with small cross-sectional area relative to that of the cavity volume, which won't be all cases. Ingard explained both ends of the neck (facing inside and outside of the resonator) must be accounted for in a more accurate end correction, which inherently affects the resonant frequency of the system. Ingard gave a few examples of how different geometrical configurations lead to different resonant frequencies:

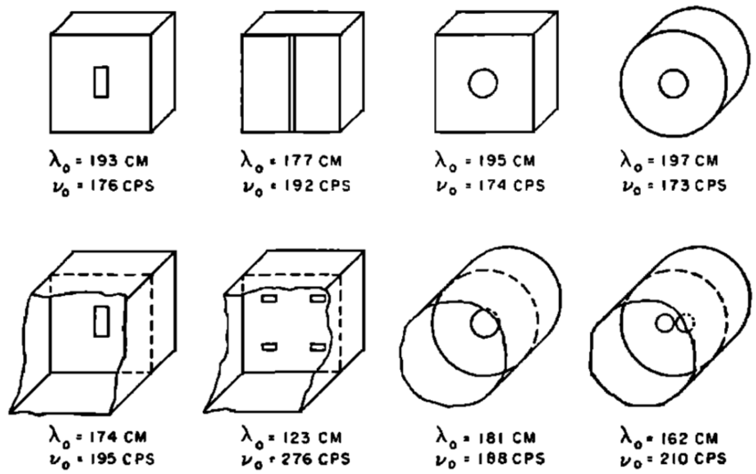

Figure 8: Varying resonant frequencies of Helmholtz resonators with the same volume and neck length, but different geometrical end corrections

Accurate calculation and prediction of a given resonator's true resonant frequency then becomes very difficult to accomplish, as this end correction must also account for the velocity profile around both ends of the aperture, which, due to non-linearities in particle velocity as a function of neck geometry, is a very difficult problem to solve analytically. Often, look-up tables are used for end corrections for general geometries. However, the fact that this is a generalization of course introduces error to any tuning or design process.

Of course, up to this point, consideration has only been given to completely empty resonators, only accounting for losses from viscous and radiation losses of the neck. If a porous absorbent material is added to the inside of the resonator, substantial increases in the absorption at resonant frequency can be achieved. When the porous layer is very near the interior or the neck, this is maximized, and falls to zero as we move to the back inner face of the acoustic cavity.

At high sound intensities, non-linear losses from turbulence occur, making accurate design of Helmholtz absorbers even more complicated. These non-linear losses are outside the scope of this paper but can be seen in [25] in their complexity. Considering variable source pressure amplitude in this situation, the interactions between the source and Helmholtz resonator will then inherently be non-linear dependent on source radiated pressure magnitude.

Slat Systems and Perforated Faces

The basic model of the Helmholtz resonator can be further expanded by

- Increasing the number of necks thereby increasing the number of acoustic “masses”, or
- Changing the baffling conditions of the “necks,” thereby changing their radiation impedance and necessary end-corrections

Here then exists a strategy to generalize the Helmholtz absorber to what is often called a “percent open area” absorber. Now, there exists one volume acts as a “spring” and many coupled masses of arbitrary count, shape, size, and baffling conditions. This absorber would then be designed by again adding these lumped elements to a circuit model and minimizing the impedance to find the resonant frequency or frequencies.

Slat systems, often consisting of wood members, are often used to this end. The wood slat members are laid on a ceiling or wall face with even or variable spacing between them which provide the apertures for the resonator. A certain depth backing the slats acts as the cavity volume and is often filled with porous, acoustically resistive material such as fiberglass.

Here, the space between the slats are the necks, and the cavity behind them is the volume. The slat system can be tuned by varying these dimensions. The Q of its absorptive peak can then be altered by increasing or decreasing the variability of the spacing and slat width. More consistent spacing will lead to a higher Q and tighter tuning, while completely random spacing and width with lead to a more broadband absorption. [26]

Perforated faces act in the same way: the air within the perforations can be thought of as a mass term and the air cavity behind them a compliance. Absorption can again be placed inside the cavity to absorb additional sound energy. Ingard showed how in addition to the absorptive material providing resistance (and thus energy dissipation) in the system, the perforated facings themselves provide resistance in some scenarios. This only occurs when the absorptive filling is within one perforation diameter from the orifice. Thus absorption can be maximized in a perforated face system when absorptive filling exists close to the orifices rather than further back in the resonant cavity. [27]

Ingard also discussed the non-linear effects of higher sound pressure level on orifices of an open area absorber (which he calls a "Helmholtz resonator array".) He demonstrated the non-linear effects through the following figures, which show the complex response for various values of β, which here is related to pressure amplitude of a sound wave normal to the orifices:

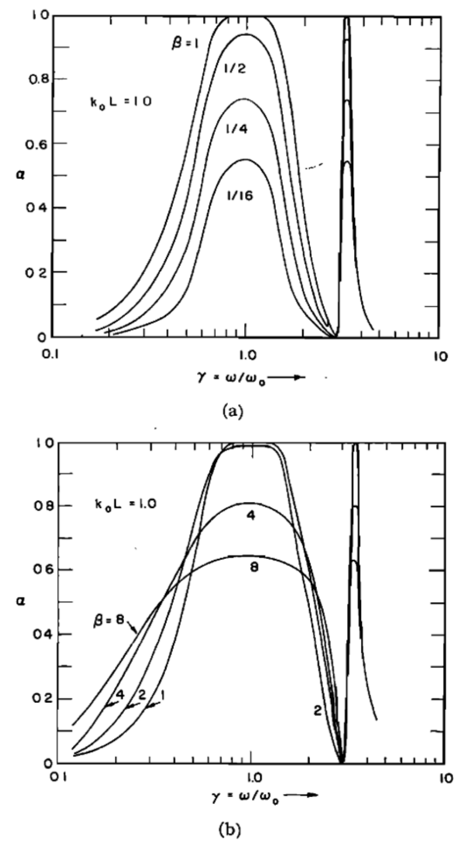

Figure 9: Absorption of Helmholtz resonator relative to face pressure amplitude factor β

It can be clearly seen that this is a complex interaction and that the width and magnitude of the resonant peak are affected by relative pressure amplitude at the face of the absorber surface.

A few measurements of both Helmholtz resonators and membrane absorbers as installed in a completely bare room can be observed in [28]. Although the microphone position in this experiment did not necessarily correspond to a realistic listening position (it is placed in the top rear corner of the space), and the absorptive treatment placement is arbitrary rather than specified on an axis related to its tuning, actual membrane and Helmholtz absorber products can be compared as installed in the field. It can be seen from the plots presented in this paper that at least the specific Helmholtz absorber used seems to not be quite as effective as some of the more effective membrane absorber units.

CHAPTER 8

SHUNTED LOUDSPEAKERS

The use of the circuit analogy has thus far been used to represent acoustical and mechanical devices but has yet to be applied to the electrical domain. Passive electrical devices such as resistors, capacitors, and inductors can be placed in series or parallel on the output of a passive loudspeaker to provide a dissipative "shunt circuit." This shunted loudspeaker can dissipate acoustic energy impinging up it through mechanical losses in the loudspeaker's reciprocal mechanical motion as well as electrical losses in the combined shunted impedance.

A circuit diagram of the loudspeaker with an arbitrary passive shunt circuit can be given as in [13], seen below

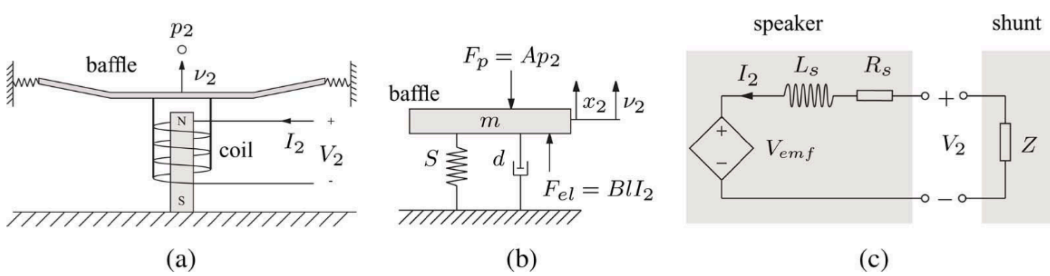

Figure 10: A loudspeaker represented in various domains. It is first represented by its physical components (a), then in the mechanical (b) and then electrical domain (c) with a shunt circuit attached to its input

It can be seen that through the mechanical damping d intrinsic to the loudspeaker, tuning of the absorption at the loudspeaker resonance with an open circuit on its output is possible. No electrical components are necessarily required for a loudspeaker to act in reverse as an acoustic absorber. Tuning could even be achieved by adding or removing mass from the face of the loudspeaker. [29]

Of course, a shunt circuit can also be added to alter the tuning and increase the loss of the system through the added electrical components. The tuning and loss of the shunted loudspeaker will be a function of its equivalent electrical impedance coupled with the acoustical and mechanical impedance of the adjoining loudspeaker system. Specific configurations of simple shunted circuits and their impedances can be seen in [13]. Difficulties in this control technique arise because the specific physical properties of the loudspeaker must be thoroughly known to achieve a given design goal. Even then, the author stated, additional experimental tweaks must be made to the system *in situ*, such as exchanging electrical or mechanical components.

Moreover, even if the ideal shunt impedance could be designed for a desired absorption response, it may not be actually realizable in an analog circuit. Regardless, using a synthetic idealized load in a reflective space, [30] is able to show significant reduction of up to 14 dB at select low-frequency modes with specific configurations, although modes below 80 Hz appear more challenging to attenuate with this given loudspeaker, presumably due to the woofer size. Lissek also demonstrated how orientation of the loudspeaker cones affect their propensity for attenuation of a given room mode—intuitively, they are most effective at attenuating modes when aligned to be normal to that mode. The benefits of a shunted loudspeaker are given by its smaller footprint in the room relative to, say, a large cavity required by a Helmholtz resonator, or the thick protrusion of a membrane absorber on the wall.

Of course, the shunt circuit of a shunted loudspeaker needn't be passive—of course, active circuits can be connected to the output terminals of a loudspeaker. Thus shunted loudspeakers bridge the gap in this discussion to active systems. [29] even argues that the simple resistance connected to the loudspeaker terminals is a "semi-active" system, since the resistor technically provides a feedback to the mechanical characteristics of the loudspeaker.

Active circuits connected to loudspeakers needn't be simple shunts, but can be fully fledged digital control systems, and could even, for example, incorporate microphone sensors for real-time signal feedback from points in the true sound field of the room. A basic example includes the general idea of an "active absorber," which has one or multiple microphones at its face and a loudspeaker acting in opposite phase to completely cancel out the impinging sound wave. Circuit and digital processing schemes for these types of techniques are outside of the scope of this paper, but the concept is clearly worth mentioning. The active control method discussion in the following section will be limited to loudspeakers connected to source content with static digital processing—that is, discussion of systems incorporating real-time measurements or controls will be forgone.

CHAPTER 9

MULTIPLE SUBWOOFERS

Optimization of the placement of the low-frequency source (which we will refer to henceforth as a subwoofer) is complicated enough, but many additional advantages can be had by installing additional sources, so the consideration of multiple sources should be strongly considered. Multiple sources can be placed in certain locations of the room and be set to variable relative phase to achieve a flatter and more regular response over the listening area. The benefit of these added sources will compound the flattening of the room's sound-field given by any passive control devices within the space.

To understand the interactions multiple subwoofers have on room modes, consideration must be given to the relative phase of the room mode pressure maxima. Odd order axial modes will have opposite phase at the boundaries and even axial modes will have identical phase—in a perfectly rigidly walled room, that is. If subwoofers excited each odd order axial mode maxima at coherent phase, the magnitude of these standing waves can be significantly reduced. [1] shows this effect by varying relative level of each subwoofer, as shown below:

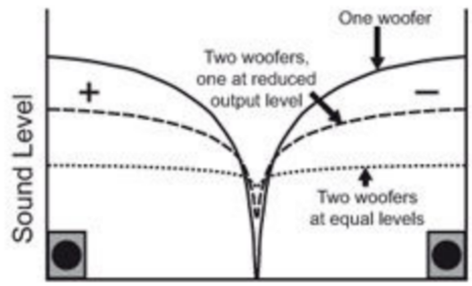

Figure 11: In-phase subwoofers exciting opposite-phase maxima of an odd order room mode

Similarly, *opposite*-phase subs set on maxima of *even* order room modes will have an identical effect, as illustrated again by Toole with the same line designations:

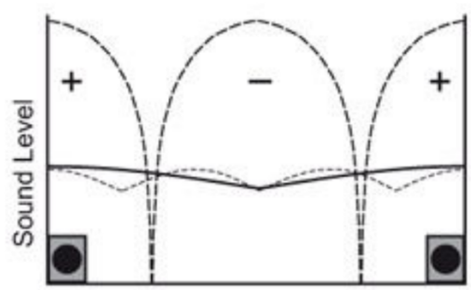

Figure 12: Opposite-phase subwoofers exciting in-phase maxima of an even order room mode

Of course, ignoring practical limits, if this methodology was applied to every single mode, we could significantly flatten the room response. In [31], Welti showed us that, should an impractically extreme amount of subwoofers be placed into a room, close to a flat modal response could be achieved. Below is his modeling of 5000 subwoofers in one simulation, with the red line representing the direct response of the subwoofer, and the black line representing the average response across the listening positions:

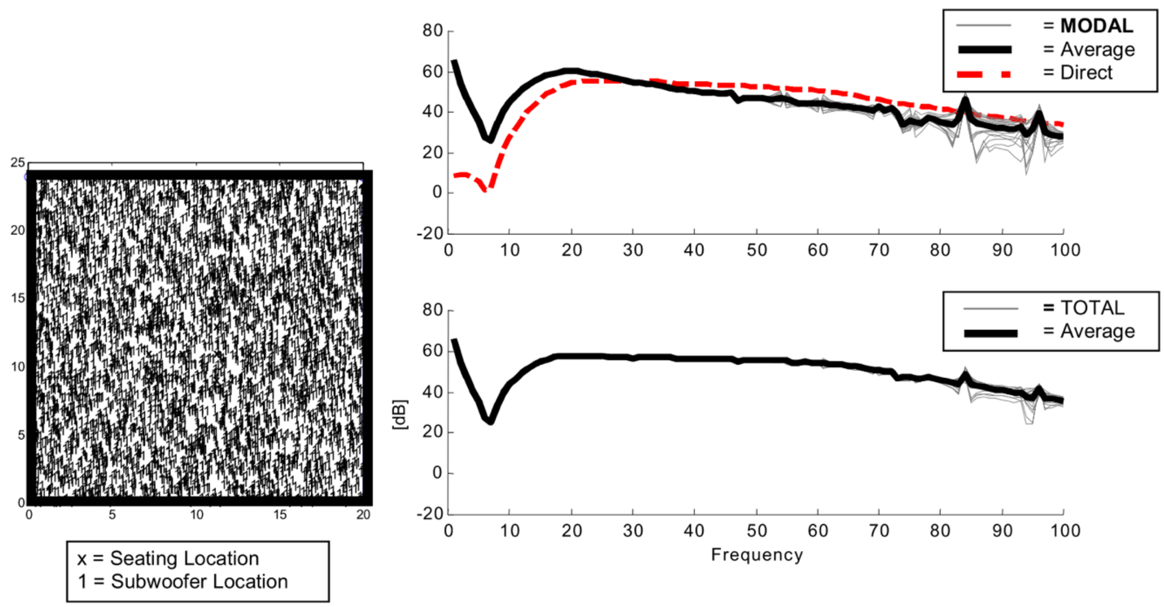

Figure 13: The effect of a large number of subwoofers on a room's frequency response

Welti modelsed and measured the frequency response of many configurations, finding the standard deviation of the individual listening area responses from the spatially averaged curve in a listening area. The idea behind his process is that, could the spatial variance of the frequency response over the listening area be minimized, any equalization applied to correct modal characteristics of the space will be more effective over a given listening area (this will be revisited in Chapter 3.2.) He modeled a certain space and came to a few conclusions for—and this is important—the case of the listening area *centered* in the room:

- Within the practical realm (i.e. with a reasonable number of subwoofers) there is no intrinsic advantage to using more than four subwoofers when their placement is optimized
- Optimal placements included
    - (4) subwoofers, each at a midpoint of a room boundary
    - (2) subwoofers at the midpoints of opposite boundaries

Care should be taken *not* to extrapolate these results to other listening area locations, as optimal placement will depend on placement of the listening area. However, this shows there *are* optimal locations for subwoofers given a set listening area.

Should there be allowance for additional variables, such as relative phase of the subwoofers, a different listening area, or even different room dimensions, things become more complicated. Additional solutions have been proposed with polarity inversions of certain subwoofers in a multiple subwoofer arrangement. One method proposed loudspeakers in the rear of the rear of the room configured to oscillate out of phase with the front speakers [32]. The subwoofers were also delayed such that they perfectly acted against the impinging sound wave from the front subwoofers, which were approximated to be acting as plane waves. This, theoretically at least, would perfectly cancel out any reflections from the rear wall, completely eliminating many modal resonances—unfortunately however it is highly dependent on precise subwoofer positioning. [33] measured a similar setup employing "source" and "sink" loudspeakers to an improved but far from perfect frequency response.

In addition to phase and delay adjustments, clearly there is the tempting option to equalize the subwoofers both globally and individually to compensate for deficiencies in the sound field by an inverted filter. This technique will be discussed in the following section.

CHAPTER 10

ROOM EQUALIZATION

When all other room mode mitigation techniques have been exhausted, the behavior of room modes can be compensated for by applying digital (or analog) filters as inversions of the problematic modal response through equalization, or EQ. The thought process is essentially that should a large modal anomaly exist in the listening position, a sharp notch or boost filter could be created to bring the level at this frequency closer to the target curve. Unfortunately, significant limitations exist for correcting both nodes and anti-nodes of room modes with EQ.

Nodal Correction Limitations

Despite the simplicity of this technique on its face, use of EQ for modal correction in practice is fairly limited. Nulls in the listening position, for example, are not practical to correct with EQ. An attempt to boost a null at the listening position back to unity gain will only result in a boost of the maximum values of the pressure response at the loudspeaker—the node of the cosine eigenfunction *must* still exist at the listening position, since the only alteration was of amplitude, not eigenfunction. Of course, complex boundary impedance will not give a true cosine eigenfunction with a standing wave null being exactly equal to zero. The “null” will in reality be greater than zero in magnitude. So, to be fair, applying a boost *will* reduce the null in a typical room, but not at all proportionally to the magnitude of the boost. The EQ boost may have to be impractically large to be effective—even then, significant distortions of the frequency response would occur in all other positions by amplifying the pressure maxima. Due to this unfortunate phenomenon, EQ is practically limited to modal *peak* mitigation.

Anti-Nodal Correction Limitations

Unfortunately, even with modal peak mitigation, solutions will be limited. [34] explains two practical constraints to room EQ: short impulse response length and a minimum-phase frequency response of the filter.

The impulse responses of room EQ filters must be short, because convolution of the output signal of the audio system and the EQ filter will lead to longer time-domain smearing with longer impulse response length. Exact inversion of the frequency response of the room in fine frequency-domain resolution—which is needed to surgically remove sharp modal peaks—results in a long impulse response. Third-octave frequency response inversions could lead to shorter impulse responses, but do not give us nearly enough resolution to deal with high-Q room modes [34].

The phase response of the filtering must also be minimum phase. Moreover, the room response must be minimum phase if inversion to the form of a practical digital filter is desired. To be minimum-phase, the z-transform of the filter's sampled (i.e. discrete) impulse response must conform to the following form:

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)}.$$

All poles and zeros of this impulse response must also lie within the unit circle in the z-domain (with exceptions to systems that can be represented as a multiplication of a minimum phase system and an allpass system [35].) Because of equivalent rules for a stable and causal time-invariant system, if the impulse response is time-invariant, stable, and causal, it will be minimum-phase and therefore invertible. Moreover, to be minimum-phase mathematically, the filter must be have the minimal phase-lag (or in other words minimum group delay) for its given magnitude response [35]. To repeat, minimum *phase* actually implies minimum *phase-lag*. As Manolakis and Ingle importantly stated, “Although the terms minimum phase-lag or minimum group delay system would be more appropriate, the term minimum phase has been established in the literature.” [35]

Thus the following is known about anti-nodal correction with EQ:

- The room mode in question must have locally minimum phase-lag to truly be invertible
    - This can be determined by finding where the *excess* group-delay from an equivalent minimum phase magnitude response is flat
- The resulting inverted filter must be minimum phase, since a non-causal filter would lead to filtered signal preceding the unfiltered signal in the time domain, which has been experimentally shown to be audible [36]. Although equivalent magnitude response filters may exist, one(s) with minimal phase-lag must be chosen.

Various axial modes may combine to produce modal characteristics that are not minimum phase. A great theoretical example is given in [37] where the interactions of three axial modes created significant non-minimum phase behavior in the response at a few points. These areas of the frequency response of the room are not treatable with EQ, but can be treated with any number of the acoustic-domain control methods mentioned in earlier chapters.

Additional Difficulties of EQ

Mitigating room modes in real rooms with EQ has additional difficulties even if minimum-phase filters can be used. Modeling of the space could be useful to predict the desired EQ, but predictive EQ techniques that are functional in practice can break down with real modal effects of finite-impedance boundary conditions. [38] outlined a method for minimizing an error function between the desired and measured complex pressure response across a listening plane in an enclosure with impressive results, but only for the case of rigid or lightly damped walls. With variable boundary impedance, predictively EQing the sources becomes difficult, since the eigenfunctions are no longer simple cosine signals.

Effective EQ then *must* rely on precise *real* measurements in the acoustic space as it is built—this is often when EQ is best applied anyway, since it is logistically simple to incorporate in the audio output chain and should be viewed as a "last resort" for low-frequency mitigation anyway. Since EQ will be applied to a listening *area* rather than a listening *point*, the listening area must be spatially sampled with a finite amount of measurements. This discrete spatial sampling limits knowledge of the true complex pressure response at every point in the plane. This brings up questions of where and how many measurements should be taken, and further, how the measurement microphone and its output varies from that which would be heard when an absorbent human body, geometrically-complex ears, and non-linear psychoacoustic effects are added to the system.

It is for these reasons that minimizing the mean spatial variance across the listening area with multiple optimally-placed sources is argued to be so important. If the frequency response is close to identical at every point on the continuum of the listening plane, any EQ will have a very similar result at each seating position within the plane. An example of the difficulty in EQing a space with highly spatially variable frequency response is given in [39].

Other techniques can be employed to minimize spatial variance of low-frequencies in a room to improve the effectiveness of global EQ of the room, including delay of the sources relative to each other, and of course equalizing each source independently such that their interaction results in a uniform sound field. Of course, this becomes extremely complicated when multiple subwoofers are used, and when their own spatial positioning is also variable.

[12] notably outlined an impressive method to minimize spatial variance of the sound field in a given room. Their algorithm attempted many combinations of various parameter values to search for the best spatial invariance in the room. The parameters they allow for each subwoofer are:

- Position
- Relative gain
- Relative delay
- A single band-stop filter per each subwoofer with variable

  - Center frequency
  - Q
  - and gain

These parameters are discretized to a few values to save computation time (e.g. gain might be in steps of 3 dB.) A search grid is made with the room modeled with every combination of parameter values, and optimal solutions are ranked. The depth of this technique shows the difficulty of providing proper room EQ at low frequencies—after all this work is done and the spatial variance is minimized, EQ must still be applied.

To review:

- EQ is difficult to predict, and therefore must be based off of measurements of the finished reproduction space
- Spatial variance must be minimal across the listening area for EQ decisions to act as improvements unilaterally across it.

Using EQ

EQ will then only have a positive effect in the case of minimum-phase modal peaks at the listening position. It will be assumed from here onwards that spatial variance across the listening area has been minimized through source positioning for simplicity of analysis.

Assuming this has been done, a few tools to manually equalize a room with real measurements can be used. First, the following metrics should be readily available:

- Spatially averaged frequency response magnitude in the listening area to determine modal peaks which can be EQ’d and modal nulls which cannot

- Excess group-delay of this frequency response to determine non-minimum phase room effects which cannot be EQ'd

These metrics could be viewed in real-time using transfer function measurements between a broadband input signal and the average of several microphones in the listening area. Quite a dense mesh of microphones should of course be used for best results.

For best results, minimal filters should of course be used to provide alterations to the measured magnitude response. Overlap of filter center frequencies should be avoided. Large frequency-spans of multiple filters may be better treated by one larger filter. For minimal audibility, boosts should be small and broad-Q, and cuts should be high-Q, precise, and can be relatively deep [1].

CHAPTER 11

CONCLUSION

As discussed, low-frequency distortions in the form of room modes are present in most music-listening environments, significantly and negatively impacting the material being presented. More unfortunate still is their complexity to control. Though all of the methods presented in this paper offer effective control of room modes, they are rather involved and ideally should be considered during the design phase of a listening space.

When designing a space for improved low-frequency reproduction, the following process may be adopted, incorporating all of the control methods presented:

1. Optimize the positioning of the subwoofer(s), listening area, and room dimensions through analysis of their interactions (most likely using predictive modeling software)
2. Correct remaining modal effects in the listening area through proper placement of membrane absorbers, Helmholtz absorbers, shunted loudspeakers, or any other low-frequency acoustic devices available.
3. Further control and final "tuning" of the space can then be refined via EQ, within the discussed limitations.

Throughout this process, both predictive modeling and real measurements must be made to ensure treatment methods having the desired effect and the sound field of the space is being improved. If adhered to, and with enough dedication, significant improvements can be made to the listening environment at low-frequencies, and the art of recorded music can be appreciated to its fullest extent.

REFERENCES

[1] F. E. Toole, *Sound Reproduction: The Acoustics and Psychoacoustics of Loudspeakers and Rooms*, 3rd ed. New York: Routledge, 2017.

[2] M. Kleiner and J. Tichy, *Acoustics of Small Rooms*, no. 1. 2014.

[3] M. R. Schroeder and M. R. Schroeder, "The '" Schroeder frequency "' revisited ɨ ɨ ɨ," *J. Acoust. Soc. Am.*, vol. 3240, no. 1996, pp. 5–7, 1996.

[4] M. R. Schröder, "Parameters of the Frequency Curves of Large Rooms," *J. Audio Eng. Soc.*, vol. 35, no. No. 5, pp. 299–306, 1987.

[5] A. Baskind and J.-D. Polack, "Sound Power Radiated by Sources in Diffuse Field," *Proc. AES 108th conv, Paris, Fr.*, vol. 5133, no. I, pp. 1–15, 2000.

[6] L. Doelle, *Environmental Acoustics*. New York, NY: McGraw Hill Company, 1972.

[7] A. D. Pierce, *Acoustics: An Introduction to Its Physical Principles and Applications*. Acoustical Society of America, 1989.

[8] D. T. Blackstock, *Fundamentals of Physical Acoustics*. New York, NY: John Wiley & Sons, Inc, 2000.

[9] F. E. Toole and S. E. Olive, "The modification of Timbre by Resonances: Perception and Measurement," *J. Audio Eng. Soc.*, vol. 36, no. 3, pp. 122–142, 1988.

[10] J. T. Du, W. L. Li, Z. G. Liu, H. A. Xu, and Z. L. Ji, "Acoustic analysis of a rectangular cavity with general impedance boundary conditions," *J. Acoust. Soc. Am.*, vol. 807, no. 2011, 2016.

[11] R. Walker, "Low-Frequency Room Responses Part 2 - Calculation methods and experimental results," 1992.

[12] T. Welti and A. Devantier, "Low-frequency optimization using multiple subwoofers," *AES J. Audio Eng. Soc.*, vol. 54, no. 5, pp. 347–364, 2006.

[13] A. J. Fleming, D. Niederberger, S. O. R. Moheimani, and M. Morari, "Control of resonant acoustic sound fields by electrical shunting of a loudspeaker," *IEEE Trans. Control Syst. Technol.*, vol. 15, no. 4, pp. 689–703, 2007.

[14] V. Knudsen and C. Harris, *Acoustical Designing in Architecture*. New York: Wiley, 1950.

[15] R. H. Bolt, "Note on Normal Frequency Statistics for Rectangular Rooms," *J. Acoust. Soc. Am.*, vol. 18, no. 1, p. 130, 1946.

[16] M. M. Louden, "Dimension Ratios of Rectangular Rooms with Good Distribution of Eigentones," *Acustica*, vol. 24, no. 2, pp. 101–104, 1971.

[17] O. J. Bonello, "A New Criterion for the Distribution of Normal Room Modes," *J. Audio Eng. Soc*, vol. 29, no. 9, pp. 597–606, 1981.

[18] T. J. Cox, P. D'Antonio, and M. R. Avis, "Room sizing and optimization at low frequencies," *AES J. Audio Eng. Soc.*, vol. 52, no. 6, pp. 640–651, 2004.

[19] P. Newell, *Recording Studio Design*, 2nd ed. 2008.

[20] A. R. Groh, "High-Fidelity Sound System Equalization by Analysis of Standing Waves," *J. Audio Eng. Soc.*, vol. 22, no. 10, pp. 795–799, 1974.

[21] T. Salava, "Subwoofers in Small Listening Rooms," *106th AES Conv.*, vol. 0, 1999.

[22] T. J. Cox and P. D'Antonio, "ROOM OPTIMIZER: A Computer Program to Optimize the Placement of Listener, Loudspeakers, Acoustical Surface Treatment and Room Dimensions in Critical Listening Rooms," *103rd AES Conv.*, 1997.

[23] T. Cox and P. D'Antonio, *Acoustic Absorbers and Diffusers: Theory, design and application*, 2nd ed. Spon Press, 2009.

[24] U. Ingard, "Transmission of Sound through a Stretched Membrane," vol. 99, no. 1954, pp. 1–4, 2016.

[25] U. Ingard, "On the Theory and Design of Acoustic Resonators," *J. Acoust. Soc. Am.*, vol. 25, no. 6, 1953.

[26] M. Rettinger, *Studio Acoustics*. CHS Press, 1981.

[27] U. Ingard, "Perforated Facing and Sound Absorption," *J. Acoust. Soc. Am.*, vol. 151, no. 1954, 2016.

[28] D. Noy, G. Hauser, and J. Storyk, "Convention Paper 5944," *115th AES Conv.*, pp. 1–8, 2003.

[29] H. Lissek, R. Boulandet, and R. Fleury, “sound absorption Electroacoustic absorbers : Bridging the gap between shunt loudspeakers and active sound absorption,” *J. Acoust. Soc. Am.*, vol. 129, no. 2968, 2011.

[30] H. Lissek, R. Boulandet, E. Rivet, and I. Rigas, “Assessment of active electroacoustic absorbers as low-frequency modal dampers in rooms,” *Proc. Internoise*, pp. 19–22, 2012.

[31] T. Welti, “How Many Subwoofers are Enough,” *112th AES Conv. Pap.*, 2002.

[32] A. Celestinos and S. B. Nielsen, “Applications of a plane wave based room correction system for low frequencies using multiple loudspeakers,” *J. Acoust. Soc. Am.*, vol. 123, no. 5, p. 3091, 2008.

[33] J. Vanderkooy, “Multi-Source Room Equalization: Reducing Room Resonances,” *123rd Conv. Audio Eng. Soc. Prepr. 7262*, 2007.

[34] L. D. Fielder, “Practical limits for room equalization,” *Audio Eng. Soc. Conv. Pap.*, p. 5481, 2001.

[35] D. Manolakis and V. Ingle, *Applied Digital Signal Processing*. 2011.

[36] L. G. Johansen and P. Rubak, “The Excess Phase In Loudspeaker / Room Transfer Functions: Can it be ignored in Equalization Tasks?,” *AES 100th Conv.*, 1996.

[37] J. Mulcahy, “Minimum Phase,” *Room EQ Wizard*. [Online]. Available: https://www.roomeqwizard.com/help/help_en-GB/html/minimumphase.html.

[38] F. Asano and D. C. Swanson, “Sound equalization in enclosures using modal reconstruction,” *J. Acoust. Soc. Am.*, vol. 98, no. 4, p. 2062, 1995.

[39] S. J. Elliot, “Practical Implementation of Low-Frequency Equalization Using Adaptive Digital Filters,” *J Audio Eng Soc*, 2013.